Chapter 2 Simultaneous Equations and Quadratics

1. Find the values of k for which the line $y = kx + 3$ is a tangent to the curve $y = 2x^2 + 4x + k - 1$. [5]

$$kx+3 = 2x^2+4x+k-1$$
$$0 = 2x^2+(4-k)x+k-4$$

$$b^2-4ac = (4-k)^2-4(2)(k-4)$$
$$= 16-8k+k^2-8(k-4)$$
$$= 16-8k+k^2-8k+32$$
$$= k^2-16k+48$$

$$\begin{matrix} - & 12 \\ - & 4 \end{matrix} \qquad 0 = (k-12)(k-4)$$

$$k = 12 \quad \text{or} \quad k = 4$$

2. Find the values of *x* for which $12x^2 - 20x + 5 > (2x + 1)(x - 1)$.

[4]

$12x^2 - 20x + 5 > 2x^2 - 2x + x - 1$

$12x^2 - 20x + 5 > 2x^2 - x - 1$

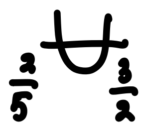
$10x^2 - 19x + 6 > 0$

$(2x - 3)(5x - 2) > 0$

2 − 3 15

5 − 2 4

$x > \frac{3}{2}$ or $x < \frac{2}{5}$



3. Find the coordinates of the points of intersection of the curves $x^2 = 5y - 1$ and $y = x^2 - 2x + 1$.

$\frac{x^2+1}{5} = y$

[5]

$\frac{x^2+1}{5} = x^2 - 2x + 1$

$x^2 + 1 = 5x^2 - 10x + 5$

$0 = 4x^2 - 10x + 4$

$\div \quad 0 = 2x^2 - 5x + 2$

2   -1   1

1   -2   4

$(2x-1)(x-2) = 0$

$x = \frac{1}{2}$ or $x = 2$

$y = \frac{1}{4}$ $y = 1$

4. (a) Write $2x^2 + 3x - 4$ in the form $a(x + b)^2 + c$, where *a*, *b* and *c* are constants.

$a(x^2 + 2bx + b^2) + c$

$ax^2 + 2abx + ab^2 + c$

$2x^2 + 3x \quad - 4$

[3]

$\therefore 2x^2 + 3x - 4 = 2(x + \frac{3}{4})^2 - \frac{41}{8}$

$a = 2$

$2ab = 3$

$4b = 3$

$b = \frac{3}{4}$

$ab^2 + c = -4$

$\cancel{2} \times \frac{9}{\cancel{16}_{8}} + c = -4$

$\frac{9}{8} + c = -4$

$c = -4 - \frac{9}{8}$

$= -\frac{41}{8}$

(b) Hence write down the coordinates of the stationary point on the curve $y = 2x^2 + 3x - 4$.

$\left(-\frac{3}{4}, -\frac{41}{8}\right)$

[2]

(c) On the axes below, sketch the graph of $y = \left|2x^2 + 3x - 4\right|$, showing the exact values of the intercepts of the curve with the coordinate axes.

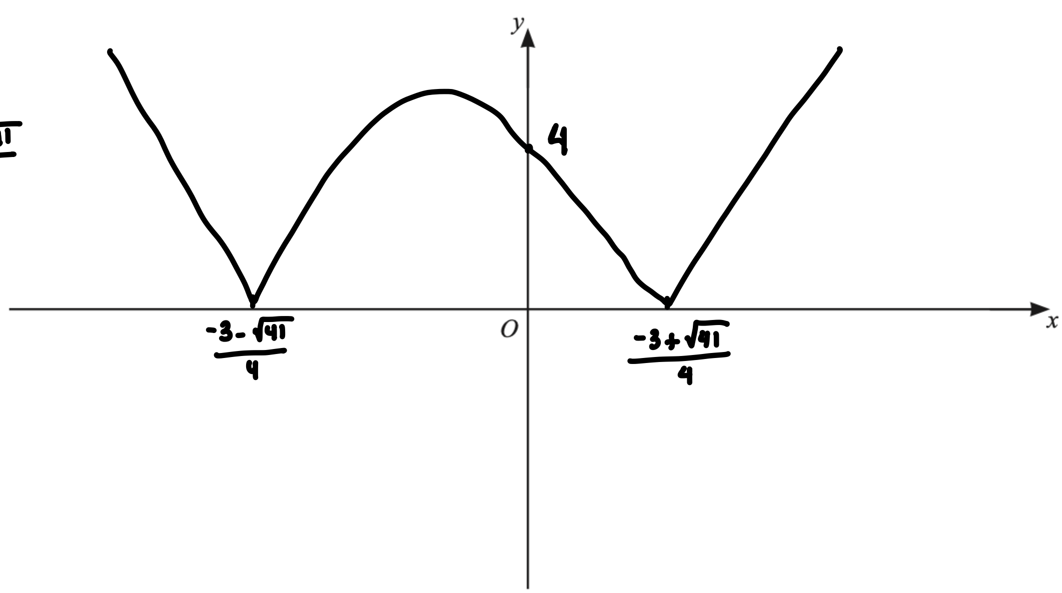
$y = 2x^2 + 3x - 4$

$x = 0, y = -4$

$y = 0, \ 2x^2 + 3x - 4 = 0$

$x = \frac{-3 + \sqrt{41}}{4}, \frac{-3 - \sqrt{41}}{4}$

[3]



(d) Find the value of *k* for which $k = \left|2x^2 + 3x - 4\right|$ has exactly 3 values of x.

$k = \frac{41}{8}$

[1]

5. (a) Write $9x^2 - 12x + 5$ in the form $p(x - q)^2 + r$, where *p*, *q* and *r* are constants.

$p(x^2 - 2qx + q^2) + r$

$px^2 - 2pqx + pq^2 + r$

$9x^2 - 12x + 5$

[3]

$p = 9$ $\quad$ $pq^2 + r = 5$

$2pq = 12$ $\quad$ $\cancel{9} \times \frac{4}{\cancel{9}} + r = 5$

$18q = 12$ $\quad$ $r = 1$

$q = \frac{2}{3}$

$9x^2 - 12x + 5 = 9(x - \frac{2}{3})^2 + 1$

checking

$9(x^2 - \frac{4}{3}x + \frac{4}{9}) + 1$

$9x^2 - 12x + 4 + 1 = 9x^2 - 12x + 5$

(b) Hence write down the coordinates of the minimum point of the curve $y = 9x^2 - 12x + 5$.

$(\frac{2}{3}, 1)$

[1]

6. Find the values of *k* for which the line $y = kx - 7$ and the curve $y = 3x^2 + 8x + 5$ do not intersect.

$kx - 7 = 3x^2 + 8x + 5$

[6]

$0 = 3x^2 + 8x - kx + 12$

$a = 3,\ b = 8 - k,\ c = 12$

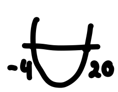
$b^2 - 4ac < 0$

$(8 - k)^2 - 4(3)(12) < 0$

$64 - 16k + k^2 - 144 < 0$

$k^2 - 16k - 80 < 0$

$(k - 20)(k + 4) < 0$



$-4 < k < 20$

7. Find the values of *k* for which the line $y = x - 3$ intersects the curve $y = k^2x^2 + 5kx + 1$ at two distinct points.

[6]

$x - 3 = k^2x^2 + 5kx + 1$

$0 = k^2x^2 + 5kx - x + 4$

$a = k^2, \; b = 5k - 1, \; c = 4$

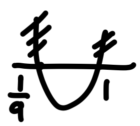
$b^2 - 4ac > 0$

$(5k-1)^2 - 4k^2(4) > 0$

$25k^2 - 10k + 1 - 16k^2 > 0$

$9k^2 - 10k + 1 > 0$

$(k-1)(9k-1) > 0$



$k > 1$ or $k < \frac{1}{9}$

8. Find the set of values of *k* for which $4x^2 - 4kx + 2k + 3 = 0$ has no real roots.

[6]

$a = 4, \; b = -4k, \; c = 2k + 3$

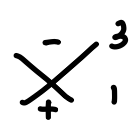
$b^2 - 4ac < 0$

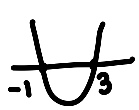
$16k^2 - 4(4)(2k+3) < 0$

$16k^2 - 16(2k+3) < 0$

$16k^2 - 32k - 48 < 0$

$k^2 - 2k - 3 < 0$





$(k-3)(k+1) < 0$

$-1 < k < 3$

9. The curve $y = 2x^2 + k + 4$ intersects the straight line $y = (k + 4)x$ at two distinct points. Find the possible values of k. [4]

$2x^2 + k + 4 = (k+4)x$

$2x^2 - (k+4)x + k + 4 = 0$

$a = 2,\ b = -(k+4),\ c = k+4$

$b^2 - 4ac > 0$

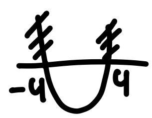
$k^2 + 8k + 16 - 4(2)(k+4) > 0$

$k^2 + \cancel{8k} + 16 - \cancel{8k} - 32 > 0$

$k^2 - 16 > 0$

$(k-4)(k+4) > 0$

$k > 4$ or $k < -4$



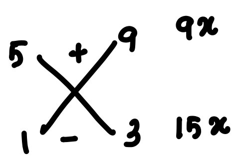
10. Find the coordinates of the points of intersection of the curve $x^2 + xy = 9$ and the line $y = \frac{2}{3}x - 2$. [5]

$x^2 + x\left(\frac{2}{3}x - 2\right) = 9$

$x^2 + \frac{2}{3}x^2 - 2x - 9 = 0$

$(\times 3)$

$3x^2 + 2x^2 - 6x - 27 = 0$



$5x^2 - 6x - 27 = 0$

$(5x+9)(x-3) = 0$

$x = -\frac{9}{5}$ or $x = 3$

$y = \frac{2}{\cancel{3}_1}\left(-\frac{\cancel{9}^3}{5}\right) - 2$ $\qquad y = \frac{2}{\cancel{3}}(\cancel{3}) - 2$

$= -\frac{6}{5} - 2$ $\qquad = 2 - 2$

$= -\frac{16}{5}$ $\qquad = 0$

$\therefore \left(-\frac{9}{5}, \frac{16}{5}\right), (3, 0)$

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11. Solve the inequality $(x - 8)(x - 10) > 35$. [4]

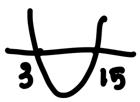
$x^2 - 10x - 8x + 80 - 35 > 0$

$x^2 - 18x + 45 > 0$

$(x-15)(x-3) > 0$

$x > 15$ or $x < 3$

-15, -3



12. Solve the simultaneous equation. [5]

$$x^2 + 3xy = 4$$

$$2x + 5y = 4$$

$5y = 4 - 2x$

$y = \frac{4-2x}{5}$

$x^2 + 3x\left(\frac{4-2x}{5}\right) = 4$

$x^2 + \frac{12x - 6x^2}{5} = 4$

$5x^2 + 12x - 6x^2 = 20$

$-x^2 + 12x - 20 = 0$

-10, -2

$x^2 - 12x + 20 = 0$

$(x-10)(x-2) = 0$

$x = 10$ or $x = 2$

$y = \frac{4-2(10)}{5}$ $\quad y = \frac{4-4}{5}$

$= \frac{4-20}{5}$ $\quad = 0$

$= \frac{-16}{5}$

$(10, -\frac{16}{5})$ $(2, 0)$

13. Find the values of k for which the equation $x^2 + (k+9)x + 9 = 0$ has two distinct real roots.

[4]

$a=1, b=k+9, c=9$

$b^2-4ac > 0$

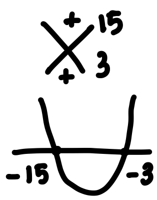
$(k+9)^2-4(9) > 0$

$k^2+18k+81-36 > 0$

$k^2+18k+45 > 0$

$(k+15)(k+3) > 0$

$k > -3$ or $k < -15$



14. $f(x) = x^2 + 2x - 3$ for $x \geq -1$

a. Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why f(x) has an inverse.

because $f(x)$ becomes a one-to-one function when $x \geq -1$

[1]

b. On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes.

[4]

$y = x^2+2x-3$

$x=0, y=-3$

$y=0, x=1$ or $x=-3$